CMX Calculations

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Table of Contents

CMX Calculations

This presentation of CMX's calculations uses terms like "*Confidence Level*", "*Coverage Factor*, "*Standard Deviation*" etc. If you are not familiar with the terms, please consult documents presenting statistical mathematics and uncertainty calculation.

Some of the calculations presented here do not apply to switches. In that case, the switch specifics are presented after the general presentation of a calculation.

Selecting What is Calculated/Shown

This chapter presents where calculation related settings are located in CMX.

The error calculation method is selected in the **Calibration Procedure** property window.

In the options window's calibration page are the following settings available (**Tools** > **Options** > **Calibration page**):

• Are the **average** calibration **results** shown or not.
• Are the calibration results' **expanded uncertainty** data shown or not.

The following setting is available in the *Function* **property** window:

• Which **repeatability formula** is in use for calculating the average results:

    1. *Standard Deviation* of Output or
    2. Standard Deviation of Average Output.

For more information, see chapter Deviation of the Output Values.

Error Calculation

The following error calculation methods are available in CMX:

- **Absolute error**
- **Percent of span**
- **Percent of reading**
- **Percent of rate** (only in older calibration procedures)
- **Percent of load**
- **Percent of Accuracy Class**

Whichever method is used, the maximum error shown among the results is the highest absolute value of all errors calculated for each calibration point.

Equations

For more information on the formulas used in Error Calculation equations, see the subsequent chapters.

Ideal Output and Input

All error calculations need to know what the "ideal value" for a reading is. Typically the instrument's input is seen as the "correct value" and the corresponding ideal output is calculated based on the input value and the transfer function as follows:

$$O_{ideal} = \left(\frac{(I - I_{zero})}{(I_{fs} - I_{zero})} \right)^t \cdot (O_{fs} - O_{zero}) + O_{zero} \tag{1}$$

Where:

I	is the measured input for a calibration point.
I_{zero}	is the theoretical input value at Input 0%.
I_{fs}	is the theoretical input value at Input 100% (full scale).

| t | is a transfer function dependent exponent: |

	t = 1	for linear transfer function,
	t = 1/2	for square root (rooting) transfer function,
	t = 2	for squared transfer function,
	t = 3/2	for "power 1.5" transfer function and
	t = 5/2	for "power 2.5" transfer function.

| O_{fs} | is the theoretical output value at Output 100% (full scale). |
| O_{zero} | is the theoretical output value at Output 0%. |

In certain situations, the output is considered as the "correct value" and then the corresponding ideal input is calculated based on the output value and the inverted transfer function as follows:

$$I_{ideal} = \left(\frac{(O - O_{zero})}{(O_{fs} - O_{zero})} \right)^{t'} \cdot (I_{fs} - I_{zero}) + I_{zero} \qquad (2)$$

Where:

O	is the measured output for a calibration point.
O_{zero}	is the theoretical output value at Output 0%.
O_{fs}	is the theoretical output value at Output 100% (full scale).
t'	is an inverted transfer function dependent exponent:

	t' = 1	for linear transfer function,
	t' = 2	for square root (rooting) transfer function,
	t' = 1/2	for squared transfer function,
	t' = 2/3	for "power 1.5" transfer function and
	t' = 2/5	for "power 2.5" transfer function.

> **i** **Note:** For CMX in the transfer function is always a conversion **from input to output**. Thus the inverted transfer function is used here for calculating the ideal input.

| I_{fs} | is the theoretical input value at Input 100% (full scale). |
| I_{zero} | is the theoretical input value at Input 0%. |

> **Note:** The equations above apply to CMX standard transfer functions. Not for any user defined custom transfer functions.

Absolute Error

Absolute error is the error presented in engineering units.

Equations:

Output error, all values are in engineering units:

$$E_{O\ abs} = O - O_{ideal} \qquad\qquad (3)$$

Where:

$E_{O\ abs}$	is the calculated absolute output error for a calibration point.
O	is the measured output for a calibration point.
O_{ideal}	is the theoretical output value at a calibration point. See chapter Ideal Output and Input.

Input projected error, all values are in engineering units:

$$E_{I\ abs} = I_{ideal} - I \qquad\qquad (4)$$

Where:

$E_{I\ abs}$	is the calculated absolute input error for a calibration point.
I_{ideal}	is the measured output value projected (using the Transfer function equation) to input, i.e. the input value that theoretically produces the measured output value. See chapter Ideal Output and Input.
I	is the measured input for a calibration point. For Weighing Instruments:
	Either Conventional Mass or Nominal Mass depending on your Calibration Procedure settings.

> **Note:**
>
> See also chapter Error Resolution, Absolute Error.

Percent of Span

Output error:

$$E_{O\ span} = \frac{O - O_{ideal}}{O_{fs} - O_{zero}} \cdot 100\%$$ (5)

Where:

$E_{O\ span}$	is the calculated output error (percent of span) for a calibration point.
O	is the measured output for a calibration point.
O_{ideal}	is the theoretical output value at a calibration point. See chapter Ideal Output and Input.
O_{fs}	is the theoretical output value at Output 100% (full scale).
O_{zero}	is the theoretical output value at Output 0%.

Input projected error:

$$E_{I\ span} = \frac{I_{ideal} - I}{I_{fs} - I_{zero}} \cdot 100\%$$ (6)

Where:

$E_{I\ span}$	is the calculated input error (percent of span) for a calibration point.
I_{ideal}	is the measured output value projected (using the Transfer function equation) to input, i.e. the input value that theoretically produces the measured output value. See chapter Ideal Output and Input.
I	is the measured input for a calibration point.
I_{fs}	is the theoretical input value at Input 100% (full scale).
I_{zero}	is the theoretical input value at Input 0%.

> *i* **Note:** See also chapter Error Resolution, Percentage Errors.

Percent of Reading

Output error:

$$E_{O\ rdg} = \frac{E_{o\ abs}}{|O_{ideal}|} \cdot 100\ \%\qquad(7)$$

Where:

$E_{O\ rdg}$	is the calculated output error (percent of reading) for a calibration point.
$E_{O\ abs}$	is the calculated absolute output error for a calibration point (see Absolute Error).
O_{ideal}	is the theoretical output value at a calibration point. See chapter Ideal Output and Input.

Input projected error:

$$E_{I\ rdg} = \frac{E_{I\ abs}}{|I_{ideal}|} \cdot 100\ \%\qquad(8)$$

Where:

$E_{I\ rdg}$	is the calculated input error (percent of reading) for a calibration point.
$E_{I\ abs}$	is the calculated absolute input error for a calibration point.
I_{ideal}	is the measured output value projected (using the Transfer Function equation) to input, i.e. the input value that theoretically produces the measured output value. See chapter Ideal Output and Input.

> **Note:** Since mathematical problems arise when the divider is zero, CMX does not calculate Percent of Reading Error if the O_{ideal} (respectively I_{ideal}) is zero. In general: Avoid using Percent of Reading error calculation if there are *Calibration Points* near zero. See also chapter Error Resolution, Percentage Errors.

Percent of Rate

Percent of Rate error calculation is no longer available for new calibration procedures. Existing procedures utilizing Percent of Rate still have this error calculation method available.

Output error:

$$E_{O\ rate} = \frac{E_{O\ abs}}{\left| O_{ideal} - O_{zero} \right|} \cdot 100\ \% \qquad (9)$$

Where:

$E_{O\ rate}$	is the calculated output error (percent of rate) for a calibration point.
$E_{O\ abs}$	is the calculated absolute output error for a calibration point (see Absolute Error).
O_{ideal}	is the theoretical output value at a calibration point. See chapter Ideal Output and Input.
O_{zero}	is the theoretical output value at Output 0%.

Input projected error:

$$E_{I\ rate} = \frac{E_{I\ abs}}{\left| I_{ideal} - I_{zero} \right|} \cdot 100\ \% \qquad (10)$$

$E_{I\ rate}$	is the calculated input error (percent of reading) for a calibration point.
$E_{I\ abs}$	is the calculated absolute input error for a calibration point.
I_{ideal}	is the measured output value projected (using the Transfer Function equation) to input, i.e. the input value that theoretically produces the measured output value. See chapter Ideal Output and Input.
I_{zero}	is the theoretical input value at Output 0%.

> **Note:** Since mathematical problems arise when the divider is zero, CMX does not calculate Percent of Rate Error if O_{ideal} - O_{zero} (respectively I_{ideal} - I_{zero}) is zero. In general: Avoid using Percent of Rate error calculation if there are *Calibration Points* near zero.
>
> See also chapter Error Resolution, Percentage Errors.

Percent of Load

This error calculation formula is used in Weighing Instruments only.

Output error:

$$E_{O\,load} = \frac{E_{O\,abs}}{\left| I_{ideal(load)} \right|} \cdot 100\ \% \qquad (11)$$

Where:

$E_{O\,load}$	is the calculated output error (percent of load) for a calibration point.
$E_{O\,abs}$	is the calculated absolute output error for a calibration point. See equation 3 in chapter Absolute Error.
$I_{ideal(load)}$	is the mass of the used weight. Use either Conventional Mass or Nominal Mass, depending on your Calibration Procedure settings

Input projected error:

$$E_{I\,load} = \frac{E_{I\,abs}}{\left| I_{ideal(load)} \right|} \cdot 100\ \% \qquad (12)$$

Where:

$E_{I\,load}$	is the calculated input error (percent of load) for a calibration point.
$E_{I\,abs}$	is the calculated absolute error projected (using the Transfer Function equation) to input. See equation 4 in chapter Absolute Error.
$I_{ideal(load)}$	is the mass of the used weight. Use either Conventional Mass or Nominal Mass, depending on your Calibration Procedure settings.

> **Note:** Since mathematical problems arise when the divider is zero, CMX does not calculate Percent of Load Error if the used load (mass) is zero. See also chapter Error Resolution, Percentage Errors.

Percent of Accuracy Class

Percent of Accuracy Class error calculation is no longer available for new calibration procedures. Existing procedures utilizing Percent of Accuracy Class still have this error calculation method available.

This error calculation formula is used in Weighing Instruments only.

Output error:

$$E_{O\,acl} = \frac{E_{O\,abs}}{|AccClassLimit|} \cdot 100\,\% \qquad (13)$$

Where:

$E_{O\,acl}$	is the calculated output error (percent of accuracy class) for a calibration point.
$E_{O\,abs}$	is the calculated absolute output error for a calibration point. See equation 3 in chapter Absolute Error.
AccClassLimit	is the Accuracy Class Limit defined in the instrument's Calibration Procedure.

Input projected error:

$$E_{I\,acl} = \frac{E_{I\,abs}}{|AccClassLimit|} \cdot 100\,\% \qquad (14)$$

Where:

$E_{I\,acl}$	is the calculated input error (percent of accuracy class) for a calibration point.
$E_{I\,abs}$	is the calculated absolute error projected (using the Transfer Function equation) to input. See equation 4 in chapter Absolute Error.
AccClassLimit	is the Accuracy Class Limit defined in the instrument's Calibration Procedure.

> **Note:** Since mathematical problems arise when the divider is zero, CMX does not calculate Percent of Accuracy Class Error if the Accuracy Class Limit of the calibration point is near zero.
>
> See also chapter Error Resolution, Percentage Errors.

About Error Resolution

CMX's **Calibration Procedure** includes an **Error Resolution** setting. It affects how error calculation results are displayed in CMX and how pass or fail status is determined for a calibration.

Depending on your error calculation method, **Error Resolution** setting should be taken into account differently. They are described in subsequent chapters.

> ⓘ **Note:** **Error Resolution** setting applies to the error results presented in tables, not to most of the error summary statistics presented below the tables. Error summary statistics data are e.g. **Maximum Hysteresis**, etc. (for standard instruments) and **Max Set Error**, etc. (for switches).

Error Resolution, Absolute Error

Less Decimals Displayed Than What is Expected.

Error calculation results presented in the calibration result tables follow **Error Resolution** setting except:

• When the resolution of the output reading is worse than the **Error Resolution** setting, then the resolution of the reading is used when presenting error calculation results.

The rule above applies to absolute error calculations when output error is calculated. If the error is calculated for the input, the resolution of the input reading is used when deciding the resolution of error calculation results.

Example of frequency instrument's calibration where **Calibration Procedure**'s **Error Resolution** is set to **0.001**:

Indication/Hz	Absolute Error (not rounded)	Absolute Error (rounded/displayed)
1.00001	0.00001	0.000
10.0002	0.0002	0.000
100.003	0.003	0.003
1000.04	*0.04*	*0.04(* *
10000.5	*0.5*	*0.5(* *

) The resolution of these error values are not as set in the **Error Resolution** field because the resolution of the frequency reading has less decimals than the suggested **Error Resolution** setting.

Error Resolution and Determining Pass/Fail

When the error is calculated/presented as **Absolute Error**, any extra measured or entered digits that are "more accurate" than the **Error Resolution** setting may cause unexpected results: First the error is calculated and then the error value is rounded to defined resolution. The rounded value is used for determining whether the calibration has passed or failed. See example in table below:

Defined error resolution:	**0.01**
Defined error limit @ calibration point:	max. ≤ **0.05**
Ideal signal @ calibration point:	**1.50**
Entered or measured signal @ calibration point. (Extra digits, compared to error resolution setting, are unbolded):	**1.55**493
Calculated absolute error before rounding (not shown):	**0.05493**
Calculated/displayed absolute error after rounding:	**0.05**
If determining pass or fail would be done from calculated absolute error before rounding:	**0.05493 > 0.05 => FAIL**
Determining pass or fail from calculated absolute error after rounding (Which is used in CMX):	**0.05 = 0.05 => PASS**

Avoid entering more digits than the defined **Error Resolution**, or adjust the **Error Resolution** setting to better suit your needs.

Error Resolution, Percentage Errors

Error calculation methods that include percentage calculation may give results that have spurious digits, i.e. extra digits at the end of the calculated error value. These are not valid data, but "random numbers" caused by the calculation.

Available error calculation methods that include percentage calculation:

- **Percent of Span,**
- **Percent of Reading,**
- **Percent of Rate,**
- **Percent of Load and**
- **Percent of Accuracy Class.**

The **Error Resolution** field in CMX's **Calibration Procedure** is a tool for cutting off spurious digits when using error calculation method including percentage. So, when defining the **Error Resolution**, think of the available significant numbers in the Input and Output data. If the **Error Resolution** setting gives results with notably different amount of numbers than the significant numbers in input and output data, you may get unreliable calibration results:

Example 1, where spurious digits affect determining pass or fail of calculated error because of the Error Resolution setting:

Defined error resolution:	**0.00001**
Defined error limit @ calibration point (% of reading):	max. ≤ **0.22**
Input signal @ calibration point (span 0.000 … 4.000):	**1.159**
Ideal output signal @ calibration point (span 4.000 … 20.00, linear instrument):	**8.636**
Actual output signal @ calibration point:	**8.655**
Calculated percent of reading error before rounding (not shown in CMX):	**0.220009264**
Calculated percent of reading error after rounding (shown in CMX):	**0.22001**
Determining pass or fail from calculated percent of reading error after rounding (Which is used in CMX):	**0.22001 > 0.22=> FAIL**

All input and output signals are given with four significant numbers. The **Error Resolution** setting allows the example's errors to be shown (and used for determining pass or fail) with five significant numbers. In this example, the fifth number – the spurious digit – is decisive when determining pass or fail.

Spurious digits should not be taken into account when determining whether a calibration has passed or failed.

Example 2, where part of the real data is ignored because of the **Error Resolution** setting:

Defined error resolution:	**0.01**
Defined error limit @ calibration point (% of span):	max. ≤ **0.04**
Input signal @ calibration point (span 0.000 … 4.000):	**1.498**
Ideal output signal @ calibration point (span 4.000 … 20.00, linear instrument):	**9.992**
Actual output signal @ calibration point:	**9.999**
Calculated percent of span error before rounding (not shown in CMX):	**0.043750**
Calculated percent of span error after rounding (shown in CMX):	**0.04**
Determining pass or fail from calculated percent of span error after rounding (Which is used in CMX):	**0.04 = 0.04 => PASS**

All input and output signals are given with four significant numbers. The **Error Resolution** allows the example's errors to be shown (and used for determining pass

or fail) with a single significant number. Valid numeric data is lost because of too rigid **Error Resolution** setting.

Error Limit Function

CMX versions preceding V2, revision 2.7 contained a single **Reject if Error** Limit. Then the Error Limit was a horizontal line in a graph were the Error versus Output (alternatively Input) is presented. See example below.
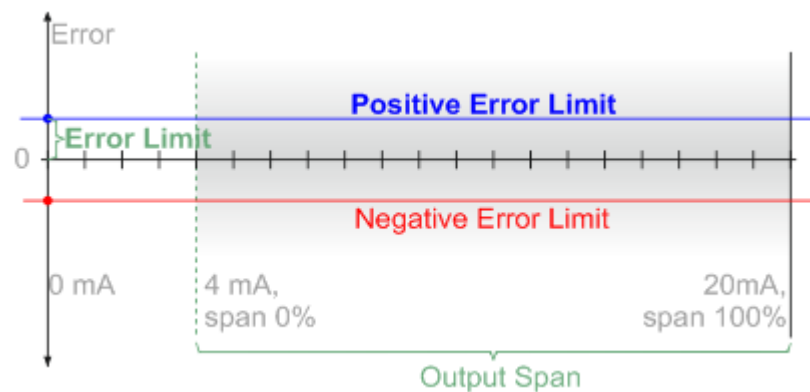
Starting from CMX V2, revision 2.7, the error can be defined as an **Error Limit Function** using two parameters:

Reject If Error (Constant) and

Reject If Error (Relative)

Reject If Error (Constant) is similar to the older **Reject if Error** Limit. Only the name has changed to separate it from the new **Reject If Error (Relative)** parameter.

Positive Error Limit Function is calculated as follows:

$$E^+_{pnt} = R_{const} + \frac{R_{rel}}{100\%} \cdot \left| RDG_{pnt} \right| \qquad (15)$$

Where:

E^+_{pnt}	Positive **Error Limit** at calibration point,
R_{const}	Reject If Error (Constant),
R_{rel}	Reject If Error (Relative), and
$[RDG_{pnt}]$	Absolute Output or Input Reading at calibration point. The Reading can be any value, even outside the Output or Input span.

Calibration Procedure setting *Error Limits Calculated From* defines whether the readings (RDG) are instrument Output or Input values. The same setting also defines if the Reject If Error (Constant) is for Output or Input.

Corresponding negative Error Limit Function is calculated from the positive Error Limit Function as follows:

$$E_{pnt}^{-} = -E_{pnt}^{+} \qquad\qquad (16)$$

The look of the Error Limit Function depends on its parameters. The following pictures present some examples and limitations.

No limitations when both

Reject If Error (Constant) and

Reject If Error (Relative)

are positive. The result is an ascending *Function*:

If
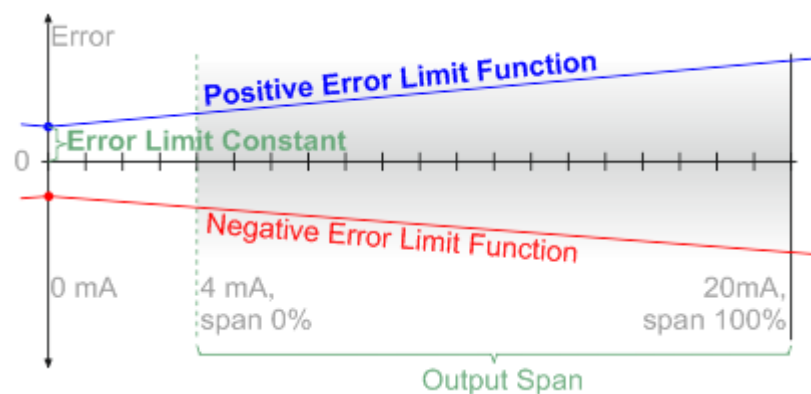
Reject If Error (Constant) is positive and

Reject If Error (Relative) is negative,

the result is a descending function. However, the Error Limit Function may not cross zero error level within the Output or Input span:

If

Reject If Error (Constant) is negative and

Reject If Error (Relative) is positive,

the result is an ascending function that starts below zero error level. However, the Error Limit Function must be above zero error level within the Output or Input span:

* Negative Error Limit Constant

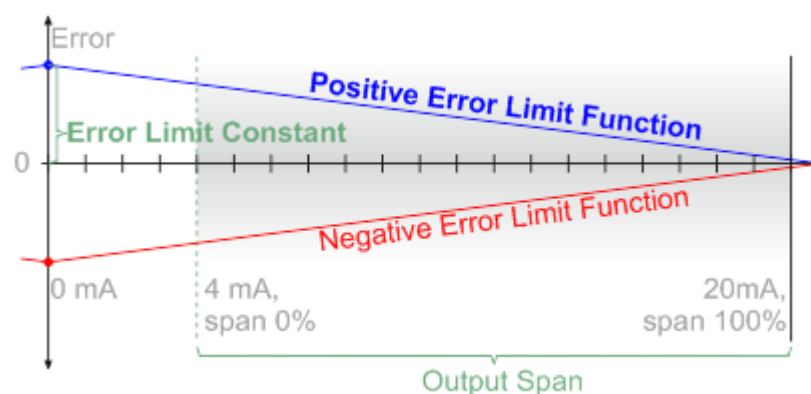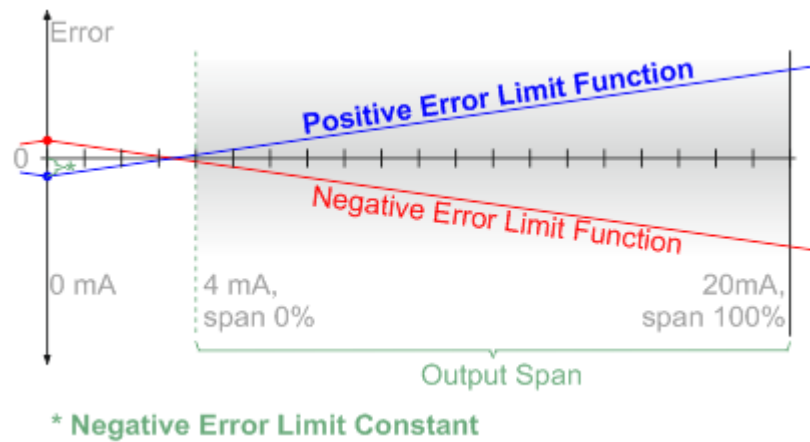Also: If **Reject If Error (Constant)** is zero, **Reject If Error (Relative)** must be positive.

Both **Reject If Error (Constant)** and **Reject If Error (Relative)** may not be negative.

Reject If Error (Relative) may be zero. Then the Error Limit Function reverts to a constant value throughout the Output or Input span.

> ℹ **Note:** When taking CMX V2, revision 2.7 in use, older instrument data using the old **Reject if Error** Limit are automatically converted so that the **Reject if Error** Limit value is "transferred" to **Reject If Error (Constant)** and **Reject If Error (Relative)** is set to zero. Thus there is no actual changes to the old instrument's error handling.

Maximum Error and Error Significance

With the use of an ascending or descending Error Limit Function, indicating the errors of *Calibration Points* require more data than with a constant value error limit.

CMX presents a **Found Maximum Error** value and a Most Significant Error as a percentage and a value, if applicable. See the picture below:
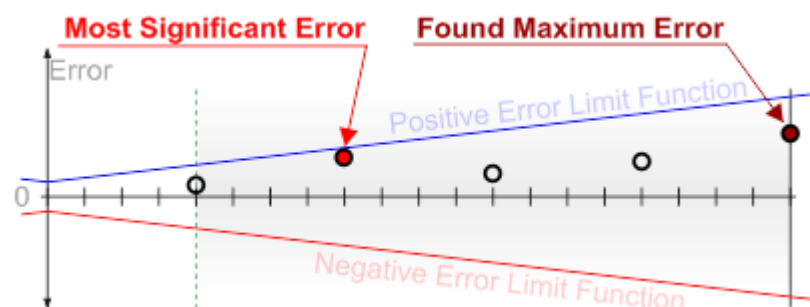
Black circles in the picture indicate calibration points and their respective errors.

Found Maximum Error (circle filled with dark red color) is the largest absolute error value in the calibration. Found Maximum Error's value is presented among calibration results.

Most Significant Error (circle filled with bright red color) is the point where the error is closest to one of the error limit functions. Most Significant Error is presented as a percentage (how close to the corresponding error limit the found error is) and as a value when it differs from the Found Maximum Error value.

Advanced Error Limits

The Error Limit can be defined with multiple subranges over the instrument's measurement range. The first subrange contains **Constant Error**, **Relative Error** and **Error Resolution**. All additional subranges, if any, contain those basic information and also a subrange **Transition point**, which is used to divide the range, as defined in How Subrange Transition Points Are Treated.

> *i* **Note:** Maximum number of subranges is unlimited in CMX, but may be limited in other Beamex products.

How Subrange Transition Points Are Treated

If an instrument has several error limit subranges and they are having separate constant error limits or slope error limit definitions, the transition point between subranges needs to be declared. For each subrange, the following data is needed:

- Subrange transition point (not needed for the first / only subrange, since in theory it starts from -∞ /+∞).

  Subrange transition points are entered and shown either as an absolute input values or as apercentages of input span.
- Error Limit's Constant Component (Constant Error), if applicable.
- Error Limit's Relative Component (Relative Error), if applicable.
- Error Resolution

Also: The first subrange starts from -∞ (+∞ for negative spans) and the last subrange continues to +∞ (-∞ for negative spans), at least in theory.

The exact transition point between subranges belongs to the subrange closer to the input span's 0 % value, e.g.:

- 1st range: to ≤ 12%
- 2nd range: >12 % to ≤ 33%
- 3rd range: >33 % to ≤ 46%
- etc.

Output values/percentages are not supported, because the error graph's x-scale is always input. Entering limits for output would cause discrepancies near a change point: output value may be on the other side of the change point than the input value

is. In that case the Error Calculation would be performed for another subrange than what is clearly seen in the error graph.

Transition point unit is shared among all subranges. If unit is configurable, the selection should be saved separately for each instrument (function).

Either Constant Component or Relative Component needs to be defined for all error limit subranges.

Error Limit Graphs for Different Error Calculation Methods

All of the following example graphs are based on following Error Limit definitions for the output signal (4 mA to 20 mA).

1st subrange:

Constant Error 1: 0.3 mA, Relative Error 1: -1.3 % of reading.

2nd subrange, starting from 50% of span:

Constant Error 2: 0.15 mA, Relative Error 2: 0.2 % of reading.

When **Error in Engineering Units** is used as the error calculation method, the graph can be calculated as presented in the functions above.
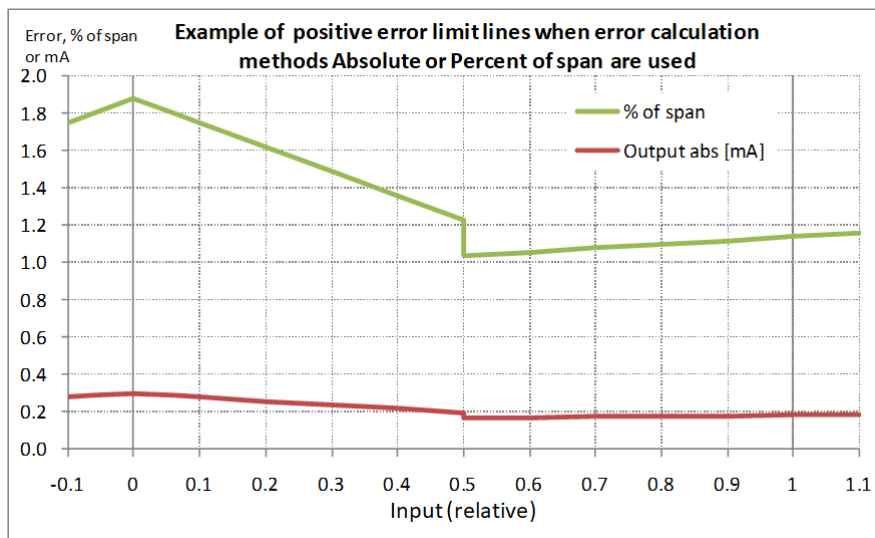
For error calculation method **Percent of Reading,** only the Relative Error is available/entered and therefore is constant within the span (subrange). Since for Relative Error's unit is "% of Reading", the error limit graph is always stepping.

Boundary Conditions for Error Limit Functions

The smallest allowed positive error limit is the value defined in **Error Resolution** setting. So if, e.g. **Error Resolution** is 0.01, the positive error limit may not be closer to zero level than 0.01. Correspondingly, the negative error limit may not be closer to zero level than -0.01.

The exception to the rules above is the first and last subrange and also when there are no subranges but a single Error Limit Function for the whole measurement span:

- The first subrange (or the only Error Limit function) may cross the zero error limit below the measurement span's 0% (zero) value. However, error limit comparisons cannot be done with input/output values smaller than where the absolute values of one or both of error limits (positive and negative) are smaller than the **Error Resolution** setting.

- The last subrange (or the only Error Limit function) may cross the zero error limit above the measurement span's 100% (full scale) value. However, error limit comparisons cannot be done with input/output values larger than where the absolute values of one or both of the error limits (positive and negative) are smaller than the **Error Resolution** setting.

If Error Limit is zero, or below zero, for an individual point, error for that point should not be calculated. Such points can cause the entire calibration to fail.

Asymmetrical Error Limits

Asymmetrical error limits mean that the positive and negative error limit lines are not symmetrical around zero error, i.e. one of the error curves is x-times further from the zero error level than the other error curve. Although multiple subranges may be used for asymmetrical error limits, the single defined asymmetry is valid throughout the measurement range. For compatibility reasons, the factor is always greater than one (>100%).

Asymmetry expansion is evaluated using the user-displayed calculated error value for the point in question, a value that is thus converted to selected error-units and also properly rounded. Asymmetry expansion requires that the calculated error for a given calibration point matches the asymmetry expansion. Error limit is in that case expanded using the factor, and rounding is performed as the final step.

Examples of variable error limits with subranges and asymmetrical error limits:
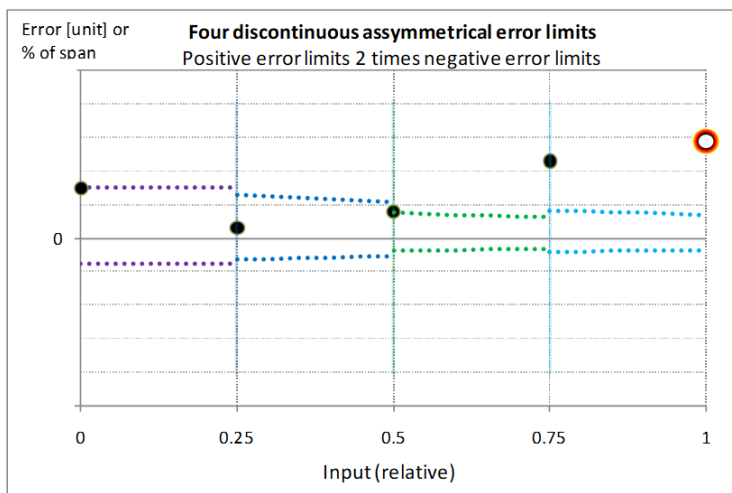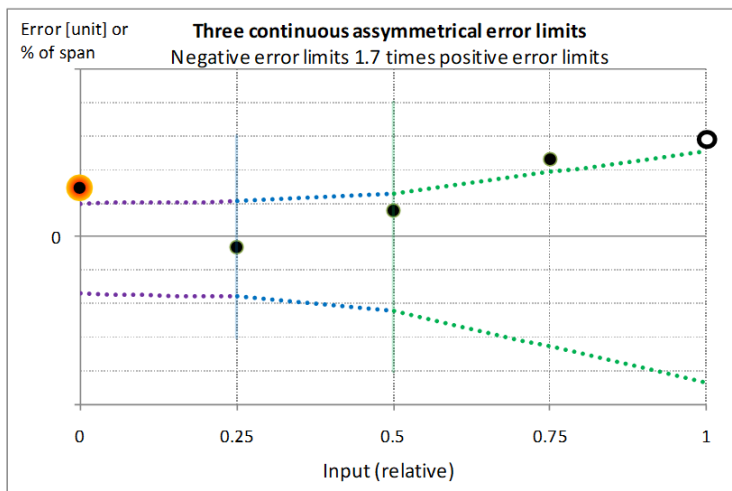
Note that the vertical lines are for illustrative purposes only. They indicate subrange transition points

Property Calculation for Non-Switches

The following calculated properties can be found among non-switch calibration results of non-switch instruments:

- **Zero Error**
- **Span Error**
- **Hysteresis**
- **Non-Conformity**

The way CMX calculates these properties is described in the subsequent chapters.

Zero and Span Error

Before counting both Zero and Span Errors, the validity of each set of upscale and downscale points is deduced.

For a set of points to be invalid, the distance between the theoretical input/output 0 % value and the point closest to it is bigger than the distance between the extreme points of that particular set of *Calibration Points*.

For all valid sets of upscale or downscale points a straight line between the extreme point error values is drawn. That line is used when extrapolating/interpolating the Zero and Span error values.
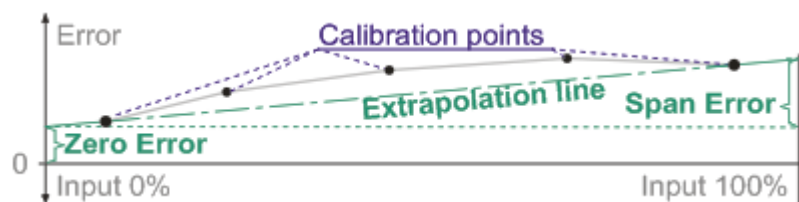
> **ℹ️ Note:** Neither Zero nor Span Error calculation is done for the first set of upscale points if there are more (and valid) upscale or downscale set of points available.

Equations:

$$\text{Slope} = \frac{E'_{fs} - E'_{zero}}{I'_{fs} - I'_{zero}} \qquad (17)$$

$$E_{zero} = \text{Slope} \cdot \left(I_{zero} - I'_{zero} \right) + E'_{zero} \qquad (18)$$

$$E_{fs} = \text{Slope} \cdot \left(I_{fs} - I'_{fs} \right) + E'_{fs} - E_{zero} \qquad (19)$$

Where:

Slope	is the calculated slope of the extrapolation/interpolation line
E'_{fs}	is the error value of the extreme point closer to Input 100% (full scale).
E'_{zero}	is the error value of the extreme point closer to Input 0%.
I'_{fs}	is the measured input value of the extreme point closer to Input 100% (full scale).
I'_{zero}	is the measured input value of the extreme point closer to Input 0%.
I_{zero}	is the theoretical input value at Input 0%.
I_{fs}	is the theoretical input value at Input 100% (full scale).
E_{zero}	is the calculated error value at Input 0%.
E_{fs}	is the calculated error value at Input 100% (full scale).

Hysteresis

The calculation of Maximum Hysteresis is divided in two parts:

- A calibration point error value being part of a set of **downscale** *Calibration Points* is interpolated to the closest next and previous **upscale** error point.
- A calibration point error value being part of a set of **upscale** calibration points is interpolated to the closest next and previous **downscale** error point.

In both cases the difference between the error value and the interpolated value is calculated. The highest absolute difference is chosen as the Maximum Hysteresis.
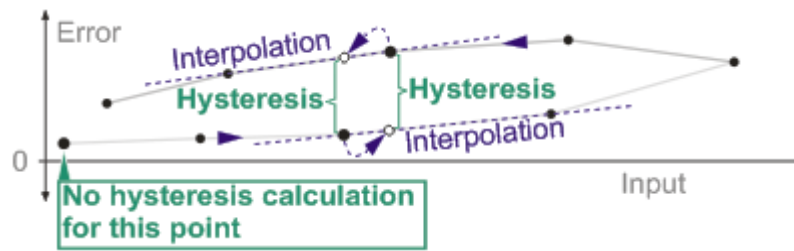
> **i** **Note:** Points where extrapolation is required to calculate the hysteresis are not taken into account.

The unit of the Maximum Error Limit also defines the display format and the unit used for Maximum Hysteresis.

Non-Conformity

For all sets of upscale or downscale points a straight line between the extreme point error values is drawn. The distance of each point error value from the drawn line is then calculated. The highest absolute value is then chosen as the Maximum Non-Conformity.
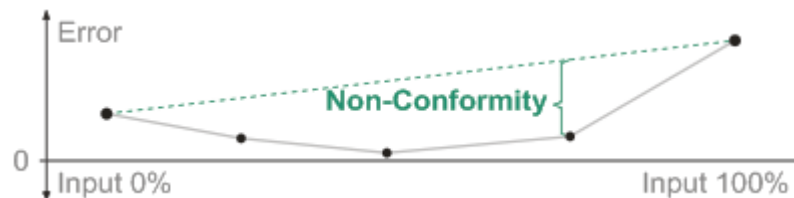
> **i** **Note:** For linear Transfer Functions, the term "Non-Linearity" is commonly used. This document uses the more general term "Non-Conformity", which is valid for all types of Transfer Functions.
>
> The type of Non-Conformity (Non-Linearity) CMX calculates is "terminal based maximum deviation from the right shape of the characteristic curve". This determines that a line between extreme point error values is used as a basis for the Non-Conformity calculation.

Average and Deviation Calculations

When doing several repeats while calibrating, CMX calculates the average values as well as deviation calculations. Whether the results are shown or not, is defined in CMX's **Options** window. For information on how to change the settings, see chapter Selecting What is Calculated/Shown.

> **Note:** In order to have as reliable average and deviation calculations as possible, the calibration repeats should be done using as similar *Calibration Points* as possible.

Switch Specifics

For switches the average values and deviation calculations can be done even if only one repeat is done, provided it includes several switch cycles.

Average Calculation

The average of corresponding *Calibration Points* of several calibration repeats is calculated using the following equation:

$$I_{avg} = \frac{\sum_{r=1}^{n} I_r}{n} \qquad (20)$$

Where

I_{avg}	is the average input value of corresponding calibration points of calibration repeats,
r	is repeat
n	is amount of corresponding points.
I_r	is the input value of repeat r.

Average output value is calculated similarly. Just replace the input values with output values. In fact any average calculation is done similarly be it average zero error, average span error or whatever.

> **Note:** The resolution of entered values greatly affect the average calculation, e.g.:
>
> - Entered data: **5** and **5.5** results in an average of: **5** (one significant number)
> - Entered data: **5.00** and **5.50** results in an average of: **5.25** (three significant number)
>
> So, be sure to enter trailing zeros that are significant to the calculations.

Switch Specifics

Average calculation applies to switches too, but instead of input and output points, the average is calculated for set and reset points respectively.

Deviation Calculation

The Deviation Calculations for Output Values and Switches are presented in the following subchapters.

Deviation of the Output Values

The deviation calculation presented here gives an approximation of the **repeatability** of the calibrated instrument. This is done using the corresponding input/output points of all repeats.

To be able to calculate the deviation of the output, all output values have to be "shifted" to correspond to the average input value. This is done as follows:

1. Using the transfer function, the ideal output value for each input value is calculated, $O_{ideal,r.}$
2. The ideal output value for the average input is also calculated, O_{Iavg}.
3. All output values are shifted to the average input value as follows: $O_{shift,r} = O_r + (O_{Iavg} - O_{ideal,r})$

Where

$O_{shift,r}$	is the "shifted" output value of a repeat,
O_r	is the recorded output value of a repeat,
O_{Iavg}	is the ideal output value for the average input value and
$O_{ideal,r}$	is the ideal output value for the repeat's input value.

The deviation calculation also requires that the average of the shifted output values of corresponding calibration points, O_{avg}, is calculated:

$$O_{avg} = \frac{\sum_{r=1}^{n} O_{shift,r}}{n} \qquad (21)$$

The repeatability formula / deviation calculation has two optional equations:

1: *Standard Deviation* **of Output / Standard deviation,** σ

$$\sigma = \sqrt{\frac{\sum_{r=1}^{n} \left(O_{shift,r} - O_{avg}\right)^2}{n-1}} \qquad (22)$$

> **Note:** This equation is always used when calculating the expanded uncertainty of each point in a calibration repeat.

2: Standard Deviation of Average Output / Standard deviation of the mean, σ_n

$$\sigma_n = \frac{\sigma}{\sqrt{n}} \qquad (23)$$

In Repeatability Formula field on function level, select one of the deviation equations for use in calculation of expanded uncertainty for each point in the table of average results. Average result showing in enabled, in options window (**Tools** > **Options** > **Calibration page**):

Select the first equation if you want the expanded uncertainty of average calibration points to include point level deviations (standard deviation) or to include the deviation of the average of repeats (standard deviation of the mean). The latter one (standard deviation of the mean) is the default equation.

Deviation Calculations of Switches

Deviation calculations for switches are done as follows:

Standard Deviation **of set point or reset point, σ**

$$\sigma = \sqrt{\frac{\sum\limits_{r=1}^{n}\left(RS_r - RS_{avg}\right)^2}{n-1}}$$ (24)

Where:

n	is the amount of repeats,
RS_r	is the set or reset point of a repeat and
RS_{avg}	is the average of set or reset points.

Standard deviation of the mean, σ_n

$$\sigma_n = \frac{\sigma}{\sqrt{n}}$$ (25)

The standard deviation is shown both for the set point and the reset point in the switch calibration average result table.

Additionally, the standard deviation of the switch's *Dead Band* is shown in the switch calibration average result tables.

Standard deviation of the dead band

$$\sigma = \sqrt{\frac{\sum\limits_{r=1}^{n}\left(DB_r - DB_{avg}\right)^2}{n-1}}$$ (26)

Where:

n	is the amount of repeats,
DB_r	is the dead band of a repeat and
DB_{avg}	is the average dead band.

Weighing Instruments

The following equations are used in Weighing Instruments only.

Pre USP 41 Calculations

The Repeatability test and Minimum Weight test calculations presented below apply when the *USP 41 Repeatability* field is **NOT** checked in Weighing Instrument's Calibration Procedure settings.

Pre USP 41 Repeatability Test

For Weighing Instruments, a Maximum Permissible *Standard Deviation* limit for a repeatability test is set in the Weighing Instrument's Calibration Procedure (field name: **Repeatability Stdev. Limit**).

The following equation calculates the found Standard Deviation for a Repeatability Test (also used in Pre USP 41 Minimum Weight Test).

$$\sigma = \sqrt{\frac{\sum_{r=1}^{n}(RDG_r - RDG_{avg})^2}{n-1}} \qquad (27)$$

Where:

n	is the amount of repeats,
RDG_r	is the reading of an individual test
RDG_{avg}	is the average of the Repeatability Test or Minimum Weight Test readings.

Pre USP 41 Minimum Weight Test

Minimum Weight Test or Minimum Sample Quantity, as it is also called, is defined in United States Pharmacopeia (USP), a quality management system in the United States, which is used primarily in the pharmaceutical industry.

According to USP, the measurement uncertainty of a Minimum Weight Test is small enough when the *Standard Deviation*, s, for ten test points times three is smaller than 0.001 compared to the weighed mass (m). Equation:

$$\frac{k \cdot \sigma}{m} \leq 0.001 \qquad\qquad (28)$$

Where:

k	is the *Coverage Factor*. Default value is 3, which is for 99 % *Confidence Level* (USP requirement before USP 41). To change the coverage factor for a particular Weighing Instrument, open the Instrument's Calibration Procedure window and look for a field called **Minimum Weight k-value**. Edit the value to suit your needs.
σ	is the Standard Deviation of the minimum weight test readings. To calculate the Standard Deviation, use formula presented in chapter Pre USP 41 Repeatability Test.
m	is the mass used in Minimum Weight Test.

> **Note:** Note that also the comparison value (0.001) is configurable in CMX (**Minimum Weight Limit** field among Procedure data). 0.001 is the default value.

USP 41 Calculations

The Repeatability test and Minimum Weight test calculations presented below apply when the *USP 41 Repeatability* field **IS** checked in Weighing Instrument's Calibration Procedure settings.

Taking USP 41 into use in CMX

When the *USP 41 Repeatability* field ischecked, the calibration of a Weighing instrument should be done with the following additional Calibration Procedure settings, required by USP 41:

• *Repeatability Accuracy Limit* need to be ≤ 0.10 % of load
• *Minimum Weight k-value* should be 2 (instead of 3 as was pre USP 41)

Also: the repeatability test has to include 10 repeats using a weight of 5 % to 100 % of the weights span.

USP 41 Repeatability Test

The *Standard Deviation* of the Repeatability test is calculated similarly as it was calculated for Pre USP 41 Repeatability Tests:

$$\sigma = \sqrt{\frac{\sum_{r=1}^{n}\left(RDG_r - RDG_{avg}\right)^2}{n-1}} \qquad (29)$$

Where:

n	is the amount of repeats,
RDG_r	is the reading of an individual test
RDG_{avg}	is the average of the Repeatability Test or Minimum Weight Test readings.

However, if the calculated Standard Deviation is smaller than 0.41·d, where d is the scale interval, the calculated Standard Deviation is replaced with **0.41·d**.

Repeatability Accuracy Limit

Repeatability test is approved if the calculated Standard Deviation (or the replaced value: 0.41·d) is smaller than the value in *Repeatability Accuracy Limit* field (default value: 0.1 % of load)

Operating Range Starting Point

Before USP 41, a separate Minimum Weight test was required. With USP 41, the repeatability test's Standard Deviation is used for calculating the Weighing Instrument's "Operating Range". The range's maximum is the Weighing Instrument maximum capacity and the range's start point is calculated as follows:

$$I_{min} = k \cdot \sigma \cdot 1000 \qquad (30)$$

Where:

I_{min}	is the operating range's start point,
k	is the value of *Minimum Weight k-value*, default value = 2.
σ	is either the repeatability test's Standard Deviation or 0.41·d, where d is the scale interval, whichever is bigger.

Additional Weights

For Weighing Instruments with digital indication, the point where the indication increases with one scale interval is determined using Additional Weights. The weights are typically 1/10 of the verification scale interval, e.

With the help of Additional Weights, the indication, P, prior to rounding can be calculated.

$$P = I + 0.5 \cdot e - \Delta L \qquad (31)$$

Where:

I	is the indication (reading) of the scale, before additional load is added.
e	is the verification scale interval.
ΔL	is the additional load.

Uncertainty Calculations

Uncertainty describes how "unsure" the measurement values are.

This section is divided into three subsections:

1. General presentation of Uncertainty Calculations and common variables (B Type Uncertainties) for all types of Uncertainty Calculations.
2. Uncertainty Calculations of Process Instruments, such as Pressure Transmitters, Temperature Indicators etc. See Uncertainty Calculations, Process Instruments.
3. Uncertainty Calculations of Weighing Instruments. See Uncertainty Calculations, Weighing Instruments.

CMX's Uncertainty Calculations of **Process Instruments** are based on the document entitled "Guide to the expression of Uncertainty in Measurement" published by ISO, IEC, BIPM, OIML and others. The abbreviation for the document is **GUM**.

CMX's Uncertainty Calculations of **Weighing instruments** are based on the document entitled "EA-10/18 Guidelines on the calibration of nonautomatic weighing instruments" published in June 2005 by **EA** (European cooperation for Accreditation).

B Type Uncertainties

First, a brief description of the other type of uncertainties, the A type uncertainties:

Some uncertainty components can be evaluated from the statistical distribution of the results of repeated measurements. These are called **A type** uncertainties.

Other uncertainty components are evaluated from assumed probability distributions based on experience or other information. They are called **B type** uncertainties. Examples of B type uncertainties:

* the calibrator's own uncertainty and
* the resolution of an indicator.

The above mentioned examples of B type uncertainties are special cases. CMX has some ready-made functionality for them:

* If a Beamex calibrator is used when calibrating an instrument, the uncertainty data of the calibrator is automatically fetched from CMX's database.
* Additionally, if no "Required Readability/Resolution" is entered, CMX calculates the resolution of an indicator based on entered data.

CMX supports entering of four additional B type uncertainties for the instrument's input and four more for the instrument's output. The additional 4 + 4 B type uncertainty data fields can be used for other B type uncertainty data, e.g. instrument's repeatability information.

Entering B Type Uncertainty Data

By default the B type uncertainty fields are not visible. To take the fields into use, open the user interface customizing tool (**Tools** > **User Interface** > **Customize**). Note that only users with the permission to customize CMX are allowed to open the window.

In the user interface customizing tool:

Open the **Calibration Result Properties** page, unhide and give a name for the B type uncertainty fields you want to use. More of User interface customizing in CMX User Guide's section **Basic Settings**, chapter **Customizing the User Interface**.

B type uncertainty data can then be entered in the **Manual Entry** window and among the calibration results (when in edit mode).

> **Note:** All B type uncertainties should be entered with 68.27 % level of confidence (standard uncertainty equaling coverage factor "1"). Otherwise the uncertainty values cannot be combined properly. Also: All added B-type uncertainties need to be uncorrelated.

Uncertainty Calculations, Process Instruments

The following Uncertainty Calculations apply to Process Instruments, such as Pressure Transmitters, Temperature Indicators etc. Uncertainty Calculations of Weighing Instruments are presented in chapter Uncertainty Calculations, Weighing Instruments.

In calibration results CMX displays both the expanded **Input Uncertainty** and the expanded **Output Uncertainty** (see The Expanded Uncertainty of a Reading) and the combined **expanded uncertainty** for the calibration point (see Expanded Uncertainty of a Calibration Point, Process Instruments).

The Expanded Uncertainty of a Reading

The results of these calculations are shown among the calibration results. See columns **Input Uncertainty** and **Output Uncertainty**.

The general equation for calculating the expanded uncertainty of a reading is as follows:

$$U_{rdg} = k \cdot \sqrt{ \sum_{mx=1}^{nx} \left(\frac{U_{calibrator, mx}}{2} \right)^2 + \left(\frac{R_{rdg}}{\sqrt{12}} \right)^2 + \sum_{j=1}^{4} \left(u_{user, j} \right)^2 } \qquad (32)$$

Where:

U_{rdg}	is the expanded uncertainty of an input or output **reading**. Headings in calibration result tables: "**Input Uncertainty**" and "**Output Uncertainty**".
k	is the *Coverage Factor* for 95.45 % level of confidence; value = 2.
$U_{calibrator, mx}$	is the expanded uncertainty of a the calibrators/ modules at that calibration point, The uncertainties are calculated based on specification data found in the following **Calibrator Module Range** fields: **Constant Error** and **Rel. Error (% of RDG)**. If the temperature during calibration was outside the temperature range of the specification (temperature range limit fields: **Spec Max. Temp.** and **Spec Min. Temp.**), then the field **Temp. Coefficient** is used for calculating an additional uncertainty term added to Ucalibrator, mx. Note that in CMX, MC6's Reference Junction specification is valid for its specified temperature range only. **Temp. Coefficient** value for MC6's Reference Junction specification is not available.
R_{rdg}	is the resolution/readability of an indicator/recorder at that calibration point. For others than indicators/ recorders, the R_{rdg} term is zero.
$u_{user, j}$	are the four user entered B type standard uncertainties for either the input or output span (if taken into use, i.e. not hidden).

The expanded uncertainties of readings are shown in the calibration result tables of each calibration repeat. Headings: "**Input Uncertainty**" and "**Output Uncertainty**".

Switch Specifics

The above mentioned equation for calculating the expanded uncertainty of a reading is also calculated for switches. The expanded uncertainty of both the set and

reset point are shown in the switch calibration result tables. Headings: "**Set Point Uncertainty**" and "**Reset Point Uncertainty**".

The Combined Standard Uncertainty of a Calibration Point

To calculate the combined standard uncertainty of a calibration point, the standard uncertainties of corresponding input and output readings must be commensurable. Therefore the input uncertainties are converted to output uncertainties or vice versa, depending on whether the uncertainty is presented for input or output (= whether CMX is configured to calculate error limits calculated from output or input).

Combined standard uncertainty is not shown in CMX, but it is a pre-phase before calculating the Expanded Uncertainty presented later on.

Combined Uncertainty Calculated for the Output

In this case all input reading uncertainties must be converted to output value uncertainties. CMX does this using the following steps:

1. An intermediate variable called Δ is calculated for each input point using the following equation:

$$\Delta = \sqrt{\sum_{mi=1}^{ni}\left(\frac{U_{icalibrator, mi}}{2}\right)^2 + \sum_{j=1}^{4}\left(u_{iuser, j}\right)^2}$$

   Where:

   $U_{icalibrator, mi}$ are the expanded uncertainties (95.45 % *Confidence Level*) of the input calibrators/modules at that calibration point. $u_{iuser, j}$ are the four user entered B type standard uncertainties for either the input or output span (if taken into use, i.e. not hidden).

2. To calculate the transfer function's slope at the proximity of an input point, ideal output values are calculated for the following input points:

   Input - Δ and

   Input + Δ

   the corresponding ideal output values are $O_{-\Delta}$ and $O_{+\Delta}$.

3. The slope, s_{io}, is calculated using the equation:

$$s_{io} = \frac{O_{+\Delta} - O_{-\Delta}}{2 \cdot \Delta}$$

> **Note:** Question:
>
> Why is CMX not using the first order derivative to calculate the slope?
>
> Answer:
>
> Using first order derivative for rooting instruments may result in division by zero. This method is more robust and it applies for all types of transfer functions.

The combined standard uncertainty of a calibration point is calculated as follows:

$$u_{co} = \sqrt{ \sigma_{(n)}^2 + \sum_{mo=1}^{no} \left(\frac{U_{ocalibrator,\,mo}}{2} \right)^2 + \cdots \left(\frac{R_{oreading}}{\sqrt{12}} \right)^2 + \sum_{j=1}^{4} \left(u_{ouser,\,j} \right)^2 + \cdots \sum_{mi=1}^{ni} \left(\frac{S_{io} \cdot U_{icalibrator,\,mi}}{2} \right)^2 + \sum_{j=1}^{4} \left(S_{io} \cdot U_{iuser,\,j} \right)^2 }$$

(33)

Where:

u_{co}	is the combined standard uncertainty of a **calibration point**, calculated for the output.
$\sigma_{(n)}$	is, depending on what is calculated (uncertainties of calibration points in a single repeat in uncertainties of average results):

- Always the *Standard Deviation* **of Output** (Standard Deviation, s) when calculating the uncertainty of each point in a calibration **repeat**.
- When calculating the uncertainty of **average** results, depending on user settings, either the **Standard Deviation of Average Output** (Standard deviation of the mean, sn) or the **Standard Deviation of Output** (Standard deviation, s). About selecting the equation: see chapter Selecting What is Calculated/Shown.

More of deviation calculations in chapter Deviation of the Output Values.

$U_{ocalibrator, mo}$	are the expanded uncertainties of the output calibrators/modules at that calibration point, The uncertainties are calculated based on specification data found in the following **Calibrator Module Range** fields: **Constant Error** and **Rel. Error (% of RDG)**. If the temperature during calibration was outside the temperature range of the specification (temperature range limit fields: **Spec Max. Temp.** and **Spec Min. Temp.**), then the field **Temp. Coefficient** is used for calculating an additional uncertainty term added to $U_{ocalibrator, mo}$.
$R_{oreading}$	is the resolution/readability of an indicator/recorder at that calibration point.
$u_{ouser, j}$	are the user entered B type standard uncertainties for the output span (if taken into use, i.e. not hidden).
s_{io}	is the slope, calculated to convert input uncertainties to output uncertainties.
$U_{icalibrator, mi}$	are the expanded uncertainties of the input calibrators/modules at that calibration point. The uncertainties are calculated based on specification data found in the following **Calibrator Module Range** fields: **Constant Error** and **Rel. Error (% of RDG)**. If the temperature during calibration was outside the temperature range of the specification (temperature range limit fields: **Spec Max. Temp.** and **Spec Min. Temp.**), then the field **Temp. Coefficient** is used for calculating an additional uncertainty term added to $U_{icalibrator, mi}$.

> **Note:** Note that in CMX, MC6's Reference Junction specification is valid for its specified temperature range only. **Temp. Coefficient** value for MC6's Reference Junction specification is not available.

$u_{iuser, j}$	are the user entered B type standard uncertainties for the input span (if taken into use, i.e. not hidden).

Other Average Result Specifics

The combined standard uncertainty of average results is calculated using the same equation as mentioned above. In addition to the exception concerning deviation calculation, the following specifics should also be noted:

- The input and output calibrator uncertainties used in calculating the combined uncertainty for average results are the calibrator's expanded uncertainties at the average input/output points.
- If the user entered B type standard uncertainties have been changed between calibration repeats, the average of each B type is used when calculating the average results.
- If one or more calibration repeats use different modules than in other calibration repeats, no average results are calculated.

Combined Uncertainty Calculated for the Input

In this case, all output uncertainties are converted to input uncertainties. The calculation procedure is similar to the previous one, but "mirrored" i.e. Δ is calculated based on output uncertainties (including possible resolution/readability of an indicator/recorder) and then used for calculating the slope s_{oi} and for converting output uncertainties to corresponding input uncertainties.

In this case, the equation for combining the uncertainties is:

$$
u_{ci} = \sqrt{
\begin{aligned}
& \sum_{mi=1}^{ni} \left(\frac{U_{ocalibrator, mi}}{2} \right)^2 + \sum_{j=1}^{4} \left(u_{iuser, j} \right)^2 + \quad \cdots \\
& \cdots \quad \left(s_{oi} \cdot \sigma_{(n)} \right)^2 + \sum_{mo=1}^{no} \left(\frac{s_{oi} \cdot U_{ocalibrator, mo}}{2} \right)^2 + \quad \cdots \\
& \cdots \quad \left(\frac{s_{oi} \cdot R_{oreading}}{\sqrt{12}} \right)^2 + \sum_{j=1}^{4} \left(s_{oi} \cdot u_{iuser, j} \right)^2
\end{aligned}
} \tag{34}
$$

Where:

u_{ci} Is the combined standard uncertainty of a **calibration point**, calculated for the input.

$U_{icalibrator,\ mi}$ Are the expanded uncertainties of the input calibrators/ modules at that calibration point. The uncertainties are calculated based on specification data found in the following **Calibrator Module Range** fields:

Constant Error and **Rel. Error (% of RDG)**. If the temperature during calibration was outside the temperature range of the specification (temperature range limit fields: **Spec Max. Temp.** and **Spec Min. Temp.**), then the field **Temp. Coefficient** is used for calculating an additional uncertainty term added to $U_{icalibrator,\ mi}$.

> *i*   **Note:** Note that in CMX, MC6's Reference Junction specification is valid for its specified temperature range only. **Temp. Coefficient** value for MC6's Reference Junction specification is not available.

$u_{iuser,\ j}$ Are the user entered B type standard uncertainties for the input span (if taken into use, i.e. not hidden).

s_{oi} Is the slope, calculated to convert output uncertainties to input uncertainties:

$$s_{oi} = 1/\ s_{io}.$$

$\sigma_{(n)}$ Is, depending on what is calculated (standard uncertainties of *Calibration Points* in a single repeat or standard uncertainties of average results):

- Always the *Standard Deviation* **of Output** (Standard Deviation, s) when calculating the standard uncertainty of each point in a calibration **repeat**.
- When calculating the standard uncertainty of **average** results, depending on user settings, either the **Standard Deviation of Average Output** (Standard deviation of the mean, sn) or the **Standard Deviation of Output** (Standard deviation, s). About selecting the equation: see chapter Selecting What is Calculated/Shown.

More of deviation calculations in chapter Deviation of the Output Values.

$U_{ocalibrator, mo}$	Are the expanded uncertainties of the output calibrators/ modules at that calibration point. The uncertainties are calculated based on specification data found in the following **Calibrator Module Range** fields:
	Constant Error and **Rel. Error (% of RDG)**. If the temperature during calibration was outside the temperature range of the specification (temperature range limit fields: **Spec Max. Temp.** and **Spec Min. Temp.**), then
	the field **Temp. Coefficient** is used for calculating an additional uncertainty term added to $U_{ocalibrator, mo}$.
$R_{oreading}$	Is the resolution of an indicator/recorder at that calibration point.
$u_{ouser, j}$	are the user entered B type standard uncertainties for the output span (if taken into use, i.e. not hidden).

Expanded Uncertainty of a Calibration Point, Process Instruments

The **Expanded Uncertainty** (to be precise, the **Expanded Uncertainty of the Error**) is shown among the calibration results of non-switches, provided the expanded uncertainty is selected to be shown in the **Options** window (Calibration settings).

To expand the combined uncertainty of a calibration point to 95.45 % *Confidence Level*, the combined uncertainty is multiplied by two (*Coverage Factor* corresponding to 95.45 % confidence level).

Thus:

$$U_E = k \cdot u_c \qquad (35)$$

Where:

U_E	is the expanded uncertainty of a calibration point.
k	is the coverage factor, value=2.
u_c	is the combined standard uncertainty of a calibration point, either uco or uci.(equation 33 or equation 34 respectively).

The results calculated with the above mentioned equation are shown in result tables for each calibration repeat, provided the expanded uncertainty is selected to be shown in the **Options** window (Calibration settings). Heading: "**Expanded Uncertainty**".

The same applies to average results if they are selected to be shown in the **Options** window (Calibration settings).

Combined Expanded Uncertainty for Switches

In switch calibration you need not combine input and output certainties, but calculate the uncertainty of multiple set and reset readings separately. This is done on repeat level, if there are several switch cycles and also for average results, if there are multiple repeats. Equation:

$$U_{crs} = k \cdot \sqrt{\sigma_{(n)}^2 + \sum_{mrs=1}^{nrs} \left(\frac{U_{rscalibrator, mrs}}{2} \right)^2 + \sum_{j=1}^{4} \left(u_{user, j} \right)^2} \qquad (36)$$

Where:

U_{crs}	is the combined expanded uncertainty of a **set** or **reset point**.
k	is the *Coverage Factor*, value=2.
$\sigma_{(n)}$	is, depending on what is calculated (combined uncertainties for switch cycles in a repeat or average results of multiple repeats):

- Always the *Standard Deviation* **of set point or reset point** (σ) when calculating the combined expanded uncertainty of switch cycles in a calibration **repeat**.

    Headings in switch calibration **repeat** tables: "**Set Expanded Uncertainty**" and "**Reset Expanded Uncertainty**". See equation 24 in chapter Deviation Calculations of Switches.

- When calculating the combined expanded uncertainty of **average** results, depending on user settings, either

    the **Standard deviation of the mean** (σ_n) or

    the **Standard deviation of set point or reset point** (σ). This setting is among *Function* data.

    Headings in switch calibration **average result** tables: "**Set Expanded Uncertainty**" and "**Reset Expanded Uncertainty**". See equation 24 and equation 25 in chapter Deviation Calculations of Switches.

More of deviation calculations in chapter Deviation Calculations of Switches.

$U_{rscalibrator,\,mrs}$ Are the expanded uncertainties of the input calibrators/ modules at that calibration point. The uncertainties are calculated based on specification data found in the following **Calibrator Module Range** fields:

Constant Error and **Rel. Error (% of RDG)**. If the temperature during calibration was outside the temperature range of the specification (temperature range limit fields: **Spec Max. Temp.** and **Spec Min. Temp.**), then the field **Temp. Coefficient** is used for calculating an additional uncertainty term added to $U_{rscalibrator,\,mrs}$.

> **Note:** Note that in CMX, MC6's Reference Junction specification is valid for its specified temperature range only. **Temp. Coefficient** value for MC6's Reference Junction specification is not available.

$u_{user,\,j}$ Are the user entered B type standard uncertainties for the input span (if taken into use, i.e. not hidden).

Uncertainty Calculations, Weighing Instruments

The following Uncertainty Calculations apply to Weighing Instruments. Uncertainty Calculations of Process Instruments, such as Pressure Transmitters, Temperature Indicators etc., are presented in chapter Uncertainty Calculations, Process Instruments.

In calibration results CMX displays the standard **Input Uncertainty** (see **Standard Uncertainty of the Reference Mass**), standard **Output Uncertainty** (see **Standard Uncertainty of the Indication**) and the combined **Expanded Uncertainty** for the calibration point (see Expanded Uncertainty of a Calibration Point, Weighing Instruments).

Standard Uncertainty of the Reference Mass

The results of these calculations are shown among the calibration results of a Weighing Instrument. See **Input Uncertainty** column.

The reference mass is all the mass placed on the load receiver of the Weighing Instrument, except the small extra additive weights possibly used to define the changing point of the indication. Those small weights are considered by correcting

the indication. Normally the reference mass consists of standard weights, but sometimes so called substitution mass is used together with the standard weights.

The standard uncertainty of the reference mass during calibration is calculated as follows:

$$u_{m\,ref} = \sqrt{\left(\sum_{w=1}^{n} u_{m\,cal,\,w} + u_{m\,subst}\right)^2 + \sum_{j=1}^{4}\left(u_{m\,user,\,j}\right)^2} \qquad (37)$$

Where:

$u_{m\,ref}$	is the standard uncertainty of the reference mass.
$\sum_{w=1}^{n} u_{m\,cal,\,w}$	is the arithmetic sum of the standard calibration uncertainties of the weights on the load receiver. Arithmetic sum instead of geometric sum is used, because there is often a strong correlation between the errors of the weights.
	Each uncertainty is calculated based on specification data found in the following **Calibrator Module Range** field (i.e. weight specifications):
	Constant Error.
	Whether the uncertainty values are given for the **Nominal Mass** or the **Conventional Mass** is defined in the instrument's Calibration Procedure. The field for this setting is **Weight Mass**.
$u_{m\,subst}$	is the standard uncertainty of used substitution load. It corresponds to weight uncertainty, so it is also arithmetically added in reference mass uncertainty. When substitution loads are used, the uncertainty is entered during calibration and then automatically taken into use in uncertainty calculation.
$u_{m\,user,\,j}$	are the user entered B type standard uncertainties for the input span (reference masses), if taken into use, i.e. not hidden.
	These uncertainties could be e.g. air buoyancy, mass change of the weights since their last calibration and forces arising from convection caused by temperature differences.
	Note that these are standard uncertainties. If the distribution of the probability is other than normal, it should be converted to normal by using the respective divider.

This uncertainty tells, that if the same reference weights and perfect measurement / indication were used, statistically 68.27 % of weighing results would fit to the limits $+u_{m\,ref}$.

The results calculated with the above mentioned equation are shown in weighing instrument calibration result tables. Heading: "**Input Uncertainty**" (as standard uncertainties).

Standard Uncertainty of the Indication

The results of these calculations are shown among the calibration results of a Weighing Instrument. See **Output Uncertainty** column.

The standard uncertainty of the indication of the Weighing Instrument during calibration is calculated as a square root of the sum of squares of all essential standard uncertainty components contributing the variability of the indication, while the force to the load cells remain unchanged:

$$u_i = \sqrt{u_0^2 + u_a^2 + u_r^2 + u_x^2 + u_h^2 + \sum_{j=1}^{4} \left(u_{i user, j}\right)^2} \qquad (38)$$

Where:

u_i	is the standard uncertainty of the indication.
u_0	is the standard uncertainty component caused by the resolution of the no load indication.
	See chapter Component: No Load Resolution.
u_a	is the standard uncertainty component caused by the resolution of the at load indication.
	See chapter Component: At Load Resolution.
u_r	is the standard uncertainty component based on the imperfect repeatability test.
	See chapter Component: Repeatability.
u_x	is the standard uncertainty component based on eccentricity test.
	See chapter Component: Eccentricity.
u_h	is the standard uncertainty component based on hysteresis calculation.
	See chapter Component: Hysteresis.
$u_{i user, j}$	Are the user entered B type standard uncertainties for output span (affecting the indication), if taken into use, i.e. not hidden.
	These uncertainties could be e.g. drift of the instrument or imperfect sensitivity, discrimination or linearity. Note that these are standard uncertainties. If the distribution of the probability is other than normal, it should be converted to normal by using the respective divider.

This uncertainty tells, that if the force to the load cell remains unchanged, 68.27 % of weighing results would fit to the limits $\pm u_i$.

The results calculated with the above mentioned equation are shown in weighing instrument calibration result tables. Heading: **"Output Uncertainty"** (as standard uncertainties).

Component: No Load Resolution

This is a component of Standard Uncertainty of the Indication of Weighing Instruments. The standard uncertainty component caused by the resolution of the no load indication is calculated as follows:

$$u_0 = k_0 \cdot \frac{d_0}{\sqrt{3}} \qquad\qquad (39)$$

Where:

u_0	is the standard uncertainty component caused by the resolution of the no load indication.
d_0	is the scale interval at no load.
k_0	is a user defined factor defining how the no load resolution uncertainty is included in uncertainty calculation. There are two calibration procedure fields for scale instruments that affect no load resolution uncertainty calculation:

No Load Resol. Uncert. and

Uncert. at No Load Point.

The **No Load Resol. Uncert.** field has two options: **d0/(2*sqrt(3))**, (then $k_0 = \frac{1}{2}$).

This is the default setting.

d0/(4*sqrt(3)), (then $k_0 = \frac{1}{4}$)

This should only be used if the instrument is approved to limit the rounding error after zero setting or tare balancing to $d_0/4$.

The **Uncert. at No Load Point** field has three options:

As with other loads.

When this option is selected, formula presented in Standard Uncertainty of the Indication is used for uncertainty calculation at no load point. This is the default setting.

No Load resolution uncertainty only.

When this option is selected, only the no load resolution uncertainty (above formula) is used for uncertainty calculation at no load point.

Zero.

When this option is selected, no uncertainty calculation is done at no load point. The no load resolution uncertainty component is still included in the uncertainty calculations of other *Calibration Points*.

Since the no load reading affects the indication of every calibration point, this uncertainty is included in every calibration point.

See also: Component: At Load Resolution.

Component: At Load Resolution

This is a component of Standard Uncertainty of the Indication. The standard uncertainty component caused by the resolution of the at load indication is calculated as follows:

$$u_a = \frac{d_a}{2 \cdot \sqrt{3}} \qquad (40)$$

Where:

u_a	is the standard uncertainty component caused by the resolution of the at load indication.
d_a	is the scale interval at load.

Component: Repeatability

This is a component of Standard Uncertainty of the Indication. The equation used for calculating imperfect repeatability uncertainty depends on the amount of weighing repeats (n) in the imperfect repeatability test:

n < 4, i.e. less than four repeats

Half of the maximum and minimum difference value is used:

$$u_r = \frac{I_{max} - I_{min}}{2} \qquad (41)$$

Where:

u_r	is the standard uncertainty component based on the imperfect repeatability test.
I_{max}	is the maximum indication during imperfect repeatability test.
I_{min}	is the minimum indication during imperfect repeatability test.

n ≥ 4, i.e. four or more repeats

Standard Deviation is used:

$$u_r = \sqrt{\frac{\sum_{j=1}^{n}\left(I_j - \overline{I}\right)^2}{(n-1)}} \qquad (42)$$

Where:

u_r	is the standard uncertainty component based on the imperfect repeatability test.
I_j	is an individual test, part of the imperfect repeatability test.
$\overline{I}$	is the average indication of the imperfect repeatability test. Equation:

$$\overline{I} = \frac{1}{n} \cdot \sum_{j=1}^{n} I_j$$

n	is the number of weighing repeats in the imperfect repeatability test.

Often, the imperfect repeatability is tested using only one load. Then the same imperfect repeatability uncertainty value is used in all *Calibration Points*.

If the imperfect repeatability test is made at several loads, the imperfect readability uncertainty values are used as follows:

- At the lowest imperfect repeatability test point and below it, the value calculated for the lowest imperfect repeatability test point is used.
- At loads between two imperfect repeatability test points, the higher imperfect repeatability uncertainty of the two adjacent imperfect repeatability test points is used. Mathematically:

$$L_{r,i} < L \leq L_{r,i+1}: \quad u_r = (u_{r,i}, u_{r,i+1})_{max}$$

- Above the highest test point, the value calculated for the highest imperfect repeatability test point is used.

Component: Eccentricity

This is a component of Standard Uncertainty of the Indication. The standard uncertainty component based on eccentricity test is calculated as follows:

$$u_x = k_x \cdot \frac{I \cdot \left| \left| I_{x,j} - I_{rp} \right| \right|_{j=1...q} \Big|_{max}}{L_x \cdot \sqrt{3}} \qquad (43)$$

Where:

u_x	is the standard uncertainty component based on eccentricity test.
k_x	is a user defined factor defining how much eccentricity affects the uncertainty.

The field for defining this factor is called **Eccentricity Uncertainty** and it can be found among Weighing Instrument's **Calibration Procedure** fields. Options:

Included by Full Value, (k_x = 1)

Included by Half Value, (k_x = ½)

Not Included, (k_x = 0)

I	is the indication of the current weighing test point.
$I_{x,j}$	is an individual indication during eccentric load test.
I_{rp}	is a reference point. The average value of Minimum and Maximum eccentricity point indication.

Equation:

$$I_{rp} = \frac{I_{x\,max} + I_{x\,min}}{2}$$

q	is the number of tested locations in the eccentricity test.
	is the maximum deviation from reference point.

$$\left| I_{x,j} - I_{rp} \atop j=1\ldots q \right|_{max}$$

L_x	is the load used in eccentricity test. To be able to get reliable uncertainty components from the eccentricity test, weights below 10 % of the full scale should not be used to when determining this standard uncertainty component.

Component: Hysteresis

This is a component of Standard Uncertainty of the Indication. The standard uncertainty component based on hysteresis calculation is as follows:

$$u_h = k_h \cdot \frac{\left| I_{up,i} - I_{down,i} \right|_{max}}{2 \cdot \sqrt{3}} \qquad (44)$$

Where:

u_h	is the standard uncertainty component based on hysteresis calculation.

k_h	is a user defined factor defining whether hysteresis is taken into account in uncertainty calculation or not.
	The field for defining this factor is called **Hysteresis in Uncertainty** and it can be found among Weighing Instrument's **Calibration Procedure** fields. Options:
	Checked, (k_h = 1, included)
	Unchecked, (k_h = 0, not included)
$I_{up, i}$	is an indication with a certain load in weighing test while the loads were increased.
$I_{down, i}$	is an indication with a certain load in weighing test while the loads were decreased.

Expanded Uncertainty of a Calibration Point, Weighing Instruments

To calculate the expanded uncertainty of a calibration point's error, the uncertainties of corresponding input and output readings must be combined. For Weighing Instruments, the input and output uncertainties are commensurable, so the combining is simple compared to the same procedure for Process Instruments. Equation for calculating the combined standard uncertainty of a Weighing Instrument's calibration point:

$$u_c = \sqrt{u_{mref}^2 + u_i^2} \qquad (45)$$

Where:

u_c	is the combined standard uncertainty of a calibration point,
$u_{m\,ref}$	is the standard uncertainty of the reference mass. (input uncertainty) and
u_i	is the standard uncertainty of the indication. (output uncertainty).

> **Note:** Since combined standard uncertainty of a calibration point is an intermediate result, it is not shown among calibration results.

To expand the combined standard uncertainty of a calibration point to 95.45 % confidence level, the combined standard uncertainty is multiplied by two (*Coverage Factor* corresponding to 95.45 % *Confidence Level*).

$$U_E = k \cdot u_c \qquad \text{(46)}$$

Where:

U_E	is the expanded uncertainty of a calibration point.
k	is the coverage factor, value=2 (95.45 % confidence level).
u_c	is the combined standard uncertainty of a calibration point (combined input and output uncertainty).

The calculated **Expanded Uncertainty** (to be precise, the **Expanded Uncertainty of the Error**) is shown among the calibration results of Weighing Instruments (column **Expanded Uncertainty**), provided the expanded uncertainty is selected to be shown in the **Options** window (Calibration settings).

User Defined Temperature Sensor Calculations

This section presents how CMX utilizes **Callendar - van Dusen**, **ITS-90** and **Factor** formulas with **P**latinum **R**esistance **T**emperature (PRT) type RTD sensors. The same equation is used both for standard sensors and user defined sensors. The only difference is that the factors of the equation are customized for user defined PRT sensors.

Callendar - van Dusen

The relationship between a Platinum Resistance Temperature (PRT) sensor's resistance and temperature can be generated from the Callendar - van Dusen equation:

Below 0 °C:

$$R_t = R_0 \cdot \left[1 + A \cdot t + Bt^2 + C \cdot (t - 100) \cdot t^3 \right] \tag{47}$$

Above 0 °C:

$$R_t = R_0 \cdot \left[1 + A \cdot t + B \cdot t^2 \right] \tag{48}$$

Where:

Rt	is the resistance in Ohms at temperature t.
R0	is the resistance in Ohms at 0 °C.
t	is the temperature in °C.
A, **B** and **C**	are customizable factors.

For standard sensors the factors **A**, **B** and **C** are already defined, but real life (reference) sensors may require slightly different factors.

> **Note:** Callendar - van Dusen equation applies to Platinum Resistance Temperature (PRT) type RTD sensors only. Other types of RTD sensors cannot be customized using this equation.

ITS-90

If your calibration certificate includes coefficients for ITS-90 deviation function (International Temperature Scale of 1990) PRT sensor, select ITS-90 as the Sensor Calculation Formula. Then you may enter the constant, R tpw, and one or more coefficients: a_x , b_x …, where the subscript x is 4 to 11, depending on the deviation formula used in the calibration.

When selecting ITS-90 as the Sensor Calculation Formula you need to define the Deviation Formula (Below Zero) only if Low Limit is < 0, otherwise only normal Deviation Formula is used. Depending on the Deviation Formula you can enter one to four coefficients.

R tpw: The custom sensor's resistance in Ohms at triple point of water 0.01 °C

For more information about ITS-90 formulas, please consult the following guides:

Guide to the Realization of the ITS-90 (www.bipm.org)

International Temperature Scale of 1990 (ITS-90)(www.nist.gov)

Factor

The third way to customize an RTD sensor (not just PRTs but all types of RTDs) is using the Factor. This applies for sensors with the same relative error throughout the sensor's span, a sensor with an "offset" in its output.

Factor: A factor that offsets the standard temperature curve selected in Sensor Type.

Sensor Type: Select a standard Sensor Type.

A simple way of doing it is taking a standard RTD sensor with a 1 ohm resistance @ 0 °C, e.g. Pt1. Then enter the real resistance @ 0 °C as the Factor.

Glossary

C

Calibration Points

> In uncertainty calculations, calibration points are the input and output signal pairs recorded during calibration.

Confidence Level

> The confidence level is the probability where a value is with a given level of certainty (percentage).

Coverage Factor

> A factor that expands the probability to a given confidence level.

D

Dead Band

> The dead band of a switch is the difference between the set point and reset point in a single test.

> Dead band is sometimes also called hysteresis or differential.

Device

> A Device is the physical instrument that can be installed into a *Position*.

F

Function

> A single task done with an instrument, e.g. temperature transmitter, pressure indicator etc. Instruments capable of performing several tasks are called multifunction instruments.

> Function Type is the description of the function (input/output quantities and ranges) and information concerning the calibration of the function (e.g. *Calibration Points* and error limits).

I

Instrument

For CMX, any Position seen on the main windows' **position Tree** are instruments. No matter what kind of *Function* the instrument contains.

P

Position

A Position is the location in a process where a *Device* can be installed. Positions often have symbolic names (tags) like TTI 215 in instrumentation drawings.

A Position can be calibrated only when a device is installed because a location cannot be calibrated.

S

Standard Deviation

Standard deviation is a measure of the spread or dispersion of a set of data. The more widely the values are spread out, the larger the standard deviation.

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